Bourbaki, Nicolas
(Éléments de mathématique. Algèbre commutative. Chapitres 1 à 4.) (French) [Zbl 1103.13001]

N. Bourbaki’s “Éléments de mathématique” has certainly been the most influential and sweeping mathematical treatise of the twentieth century. Through their monumental work, the Bourbaki group brought a revolutionary new vision of pure mathematics, a profound reorganization and clarification of its components, a unified terminology and notation, a distinctive style, and – above all – a qualitatively new degree of abstraction into mathematics as a whole. The Bourbaki members created, over a period of about sixty years, an architecture of (pure) mathematics whose compact consistency was based on three key principles: the unity of mathematics, the abstract axiomatic method, and the rigorous study of hierarchies of abstract mathematical structures. Bourbaki’s typical “structuralism” in mathematics is the predominant characteristic of the entire treatise “Elements of Mathematics”, in which the presentation is purely axiomatic and usually progresses from general to specific, without concrete examples motivating the abstract concepts. Sparse examples generally follow the abstract theoretical discussion, or are left as (sometimes very difficult) exercises.

From the pedagogical point of view, this is certainly not the most efficient method, and only a sufficiently experienced reader will be able to use any of these fundamental volumes as a supplementary textbook. Instead of ordinary textbooks, Bourbaki’s volumes must be seen as profound, conceptual source books of some of the most important theories in modern mathematics, whose significance will remain undiminished for many more decades in our 21st century, too. The first seven chapters of Bourbaki’s “Commutative Algebra” appeared between 1961 and 1965, that is right in the golden age of the structural revolution in mathematics, especially in algebraic geometry. Two more chapters were published in 1983. While the first seven chapters of Bourbaki’s “Commutative Algebra” have been reprinted several times during the past forty years, both the French originals and their translation into English (Zbl 0902.13001), the later Chapters 8 and 9 (Zbl 0579.13001) only existed as their original French edition from more than twenty years ago, and thus remained somewhat isolated from their companions.

The present new edition of Bourbaki’s “Commutative Algebra” finally incorporates all the nine chapters in their original French version. Subdivided into three handy paperback volumes, this complete collection provides, in coherent form, the unaltered reprints of the editions from the 1980s, which thereupon have been made generally available in their entirety.

The book under review is the first volume of this collection and contains the Chapters 1–4. The contents of these chapters have been reviewed utmost competently, in-their proper time, by one of the grandmasters of commutative algebra, namely by M. Nagata (Zbl 0108.04002 and Zbl 0119.03603). May it therefore suffice to just recall the topics treated in those first four chapters: 1. Flat modules; 2. Localization; 3. Gradings, filtrations and topologies; 4. Associated prime ideals and primary decomposition.

Reviewer: Werner Kleinert (Berlin)

MSC:

13–01 Introductory exposition (textbooks, tutorial papers, etc.) pertaining to commutative algebra
13C11 Injective and flat modules and ideals in commutative rings
13B30 Rings of fractions and localization for commutative rings
13A02 Graded rings
13Exx Chain conditions, finiteness conditions in commutative ring theory
13Jxx Topological rings and modules
13A15 Ideals and multiplicative ideal theory in commutative rings

Keywords:
textbook; commutative rings; ideals; flat modules; projective modules; localization; local rings; graded
rings; complete rings; filtered modules; primary decomposition

Full Text: DOI