Applications of Fukaya categories to symplectic topology require an algebraic model for these categories; this involves finding a collection of Lagrangians which generate the category in the sense that the Fukaya category fully faithfully embeds in the category of modules over the corresponding $A_\infty$ algebra. For closed symplectic manifolds, the known strategies for understanding such categories of modules rely on realising them, in an instance of homological mirror symmetry, as modules over the endomorphism algebra of (complexes of) coherent sheaves on an algebraic variety, or a non-commutative deformation thereof. Such descriptions are possible in a limited class of examples. The author is interested in the family Floer program which is to both give a more compelling proof of these equivalences, and extend the class of examples for which they can be proved. We shall call the symplectic side the $A$-side, and the algebro-geometric side the $B$-side. There are essentially only two previous results on family Floer cohomology, given by K. Fukaya and J. Tu.

The paper under review extends the author’s ICM address [“Family Floer cohomology and mirror symmetry”, Proc. Int. Congr. Math., 813–836 (2014)] by (1) constructing a map of morphism spaces from the $A$-side to the $B$-side, (2) constructing a map of morphism spaces from the $B$-side to $A$-side, (3) showing that the composition of these two maps is the identity on the $A$-side, leading to the main result of the paper under review, and (4) constructing an $A_\infty$ functor. The formal result (which is the main theorem of the article) is the following:

Theorem: Let $X \to Q$ be a Lagrangian torus fibration with $\pi_2(X) = 0$, and $L$ and $L'$ Lagrangians which are tautologically unobstructed. Given a sufficiently fine cover of $Q$, we can associate to $L$ and $L'$ (twisted) sheaves $\mathcal{F}(L)$ and $\mathcal{F}(L')$ of perfect complexes (with respect to the induced cover of $Y$), as well as maps $CF^*(L, L') \xrightarrow{\mathcal{C}} \text{Hom}(\mathcal{F}(L), \mathcal{F}(L')) \xrightarrow{\mathcal{P}} CF^*(L, L')$ whose composition is homotopic to the identity up to sign. Given a finite collection of Lagrangians, the map $\mathcal{C}$ extends to a faithful $A_\infty$ functor from the corresponding Fukaya category to the category of twisted sheaves of perfect complexes.

Reviewer: Andrzej Szczepański (Gdańsk)

MSC:

53D40 Symplectic aspects of Floer homology and cohomology
53D12 Lagrangian submanifolds; Maslov index
14G22 Rigid analytic geometry

Keywords:

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