Geisser, Thomas; Schmidt, Alexander
Tame class field theory for singular varieties over finite fields. (English) [Zbl 1386.19009]

The authors pursue their investigation of the tame class field theory for singular varieties over certain types of fields: algebraically closed fields in [Doc. Math., J. DMV 21, 91–123 (2016; Zbl 1354.14033)], finite fields here. Recall that K. Kato and S. Saito [Ann. Math. (2) 118, 241–275 (1983; Zbl 0562.14011)] generalized the class field theory for smooth, projective curves over finite fields by the “enlarged”¹ $\Pi^1_{\text{ab}}(X)$ whose images in the absolute Galois group of $\mathbb{F}$ are integral Frobenius powers ($W$ stands for Weil). If $X$ is not necessarily projective but still smooth, A. Schmidt and M. Spieß [J. Reine Angew. Math. 527, 13–36 (2000; Zbl 0961.14013); A. Schmidt [Algebra Number Theory 1, No. 2, 183–222 (2007; Zbl 1184.19002)] showed that an analogous reciprocity isomorphism $r_X : H^0_0(X, Z) \sim \pi^1_{\text{ab}}(X)_W$ holds when replacing the Chow group by the 0-th Suslin homology group, and the fundamental group by its tame version. This does not extend to non smooth schemes, for which $r_X$ is generally neither injective nor surjective.

In the present paper, the authors show that the Schmidt-Spiess result can be generalized to singular varieties on replacing the Suslin homology $H^0_0(X, Z)$ by its refined Weil-Suslin version $H^0_{\text{WS}}(X, Z)$, and the usual abelian tame fundamental group $\pi^1_{\text{ab}}(X)$ by the “enlarged”¹ $\Pi^1_{\text{ab}}(X)$ (SGA3, X, §6), of which $\pi^1_{\text{ab}}(X)$ is the profinite completion. In more detail, for any connected scheme $X$, separated and of finite type over a finite field $\mathbb{F}$, they prove two main results:

1) The pro-group $\Pi^1_{\text{ab}}(X)_W$ is isomorphic to a (constant) finitely generated abelian group.
2) There exists a refined reciprocity map $\text{rec}_X : H^0_{\text{WS}}(X, Z) \to \Pi^1_{\text{ab}}(X)_W$ which is surjective, and such that the composite with the natural map $H^0_{\text{WS}}(X, Z) \to H^1_{\text{WS}}(X, Z)$ is the previous reciprocity map $r_X$. The kernel of $\text{rec}_X$ contains the maximal divisible subgroup of $H^1_{\text{WS}}(X, Z)$, and is equal to it if resolution of singularities for schemes of dimension $\leq 1 + \dim X$ holds over $\mathbb{F}$. As a corollary, one obtains an isomorphism and $H^1_{\text{WS}}(X, Z)$ is finitely generated, hence $\text{rec}_X$ would be an isomorphism.

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11G45 Geometric class field theory
14G15 Finite ground fields in algebraic geometry

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References:


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