Geisser, Thomas; Schmidt, Alexander
Tame class field theory for singular varieties over finite fields. (English) Zbl 1386.19009

The authors pursue their investigation of the tame class field theory for singular varieties over certain types of fields: algebraically closed fields in [Doc. Math., J. DMV 21, 91–123 (2016; Zbl 1354.14033)], finite fields here. Recall that K. Kato and S. Saito [Ann. Math. (2) 118, 241-275 (1983; Zbl 0562.14011)] generalized the class field theory for smooth, projective curves over finite fields \( \mathbb{F} \) to smooth, projective varieties \( X \) of arbitrary dimension by constructing a reciprocity isomorphism \( r_X : H^0_{\text{ch}}(X) \rightarrow \pi_1^{\text{ab}}(X)_W \) between the Chow group of zero cycles and the subgroup of elements of \( \pi_1^{\text{ab}}(X) \) whose images in the absolute Galois group of \( \mathbb{F} \) are integral Frobenius powers (\( W \) stands for Weil). If \( X \) is not necessarily projective but still smooth, A. Schmidt and M. Spieß [J. Reine Angew. Math. 527, 13-36 (2000; Zbl 0961.14013); A. Schmidt [Algebra Number Theory 1, No. 2, 183–222 (2007; Zbl 1184.19002)] showed that an analogous reciprocity isomorphism \( r_X : H^S_0(X, Z) \rightarrow \pi_1^{\text{ab}}(X)_W \) holds when replacing the Chow group by the 0-th Suslin homology group, and the fundamental group by its tame version. This does not extend to non smooth schemes, for which the kernel of \( r_X \) is generally neither injective nor surjective.

In the present paper, the authors show that the Schmidt-Spiess result can be generalized to singular varieties on replacing the Suslin homology \( H^S_0(X, A) \) by its refined Weil-Suslin version \( H^{WS}_0(X, A) \), and the usual abelian tame fundamental group \( \pi_1^{\text{ab}}(X) \) by the “enlarged” \( \Pi^{\text{ab}}_1(X) \) (SGA3, X, §6), of which \( \pi_1^{\text{ab}}(X) \) is the profinite completion. In more detail, for any connected scheme \( X \), separated and of finite type over a finite field \( \mathbb{F} \), they prove two main results:

1) The pro-group \( \Pi^{\text{ab}}_1(X)_W \) is isomorphic to a (constant) finitely generated abelian group.

2) There exists a refined reciprocity map \( \text{rec}_X : H^1^{WS}(X, Z) \rightarrow \Pi^{\text{ab}}_1(X)_W \) which is surjective, and such that the composite with the natural map \( H^0_{\text{ch}}(X, Z) \rightarrow H^1^{WS}(X, Z) \) is the previous reciprocity map \( r_X \). The kernel of \( \text{rec}_X \) contains the maximal divisible subgroup of \( H^1^{WS}(X, Z) \), and is equal to it if resolution of singularities for schemes of dimension \( \leq 1+\dim X \) holds over \( \mathbb{F} \). As a corollary, one obtains an isomorphism of profinite completions \( \text{rec}_X^\wedge : H^{WS}_1(X, Z)^\wedge \rightarrow \pi_1^{\text{ab}}(X)^\wedge \). Under Parshin’s conjecture, \( H^{WS}_1(X, Z) \) is finitely generated, hence \( \text{rec}_X \) would be an isomorphism.

Reviewer: Thong Nguyen Quang Do (Besançon)

MSC:
19F05 Generalized class field theory (K-theoretic aspects)
11G45 Geometric class field theory
14G15 Finite ground fields in algebraic geometry

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References:


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