Author’s abstract: We prove two Kawamata-Viehweg type vanishing theorems with coefficients, one for polarisable variations of Hodge structures (PVHSs) and the other for general mixed Hodge modules. They generalise previous vanishing theorems of Kawamata, Viehweg, Esnault and Viehweg, Illusie, Saito, and Popa. The two theorems are complementary. One applies to all mixed Hodge modules, but only covers (of necessity) the lowest Hodge graded piece. The other applies only to Deligne’s canonical extension of PVHSs across simple normal crossings divisors, but covers all the Hodge graded pieces.

As application, we obtain new results on the cohomology of Shimura varieties and an alternative proof of a known result in birational geometry: (i) Using the one for PVHSs, we obtain new vanishing theorems for the cohomology of arbitrary Shimura varieties (including the ones attached to exceptional groups) with automorphic coefficients, extending works of Faltings, Li and Schwermer, and our own with Lan. (ii) The vanishing theorem for mixed Hodge modules contains, as a special case, Kollár’s vanishing theorem for the higher direct images of dualising sheaves and big and nef line bundles.

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MSC:

14F17 Vanishing theorems in algebraic geometry
14G35 Modular and Shimura varieties
14D07 Variation of Hodge structures (algebro-geometric aspects)

Keywords:
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