
For a complex affine algebraic variety $W$, acted upon by a reductive group $G$, a choice of character $\theta$ determines a GIT quotient $W/\!/G$. For $\varepsilon \in \mathbb{Q}_{>0} \cup \{0, \infty\}$ stability conditions produce relatively proper Deligne-Mumford moduli stacks of $\varepsilon$-stable quasimaps from pointed curves of genus $g$ to $W/\!/G$, which determine $\varepsilon$-quasimap descendant invariants when the target is projective or quasi-projective with a nice torus action. For $\varepsilon \in (2, \infty)$ they coincide with the Gromov-Witten invariants of $W/\!/G$.

The paper extends the authors’ previous conjectures on the wall-crossing formulas for these invariants to higher genus for semi-positive triples $(W, G, \theta)$ (i.e. with nef anti-canonical class), and proves them for toric varieties, including the Calabi-Yau case. The proof is inspired by Marian-Oprea-Pandharipande proof [A. Marian et al., Geom. Topol. 15, No. 3, 1651–1706 (2011; Zbl 1256.14057)] of the conjectures for $W/\!/G$ being the Grassmannian, but introduces several new ideas.

Let $J_0$, $J_1$ be the $q$-truncations of the Givental’s functions $I_0$, $I_1$: $t(\psi) = t_0 + t_1 \psi + t_2 \psi^2 + \ldots$, where $t_j \in H^*(W/\!/G, \mathbb{Q})$ are the even cohomology classes, and $F_g^\beta = \sum q^\beta/m!t(\psi_1, \ldots, t(\psi_m))_{q,m,\beta}$, $\beta \in \text{Eff}(W, G, \theta)$, be the descendant potential. Then a wall-crossing formula asserts that $(J_0^2)^{g-2}F_g^\beta(J_0^2t(\psi) - J_1^2)$ remains unchanged for all $\varepsilon$. The conjecture is proven first for $g = 0$ when $W$ admits a torus action commuting with the action of $G$, and such that the fixed points of the induced action on $W/\!/G$ are isolated.

The main result is that the conjecture holds for non-singular quasi-projective toric semi-positive GIT quotients $X = \mathbb{C}^{m+r}/(\mathbb{C}^*)^r$, and for the total spaces of the canonical bundles over type A partial flag manifolds. This implies, in particular, that the invariants of non-singular projective semi-positive Fano varieties are independent of $\varepsilon$. For the toric Calabi-Yau, $F_g^\beta|_{t(\psi) = 0}$ is the A-model genus $g$ potential after applying the mirror map. Assuming mirror symmetry, it should match the B-model potential expanded around the large complex structure point.

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MSC:

14D20 Algebraic moduli problems, moduli of vector bundles
14D23 Stacks and moduli problems
14N35 Gromov-Witten invariants, quantum cohomology, Gopakumar-Vafa invariants, Donaldson-Thomas invariants (algebro-geometric aspects)

Keywords:

stability conditions; quasimap descendant invariants; Gromov-Witten invariants; wall-crossing; toric varieties; semi-positive GIT quotients; mirror symmetry

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References:
