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Thom polynomials in $A$-classification I: counting singular projections of a surface. (English) [Zbl 1408.14173]


The authors use the theory of Thom polynomials of right-left singularities to prove enumerative results on the points of a variety $X$ in projective space that have prescribed contact with a line.

Right-left singularities are obtained by considering map germs from $m$-dimension to $n$-dimension up to holomorphic re-parametrization of the source and the target. Consider a family of such germs. Thom polynomials are universal formulas for the number of (or cohomology class represented by) points in the family where the germ belongs to a given singularity. Thom polynomials are rather well known for so-called K-singularities, but not much is known for right-left singularities. Hence, the authors use the interpolation method to calculate the Thom polynomials of certain right-left singularities for maps between low dimensional spaces.

The second part of the paper is devoted to geometric applications, as follows. Consider a variety $X^m$ in $\mathbb{P}^{n+1}$. The notion “a line $l$ having prescribed local contact type with $X$” is re-phrased as a certain germ from $\mathbb{C}^m$ to $\mathbb{C}^n$ having a particular right-left singularity. Then, the Thom polynomial formulas developed in the first half of the paper are used to calculate the number or rather the degree of such loci. For smooth surfaces in $\mathbb{P}^2$ classical formulas are rediscovered, and are generalized to singular surfaces. New formulas are proved for surfaces in $\mathbb{P}^3$ and 3-folds in $\mathbb{P}^4$, in similar spirit.

For the entire collection see [Zbl 1382.14002].

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MSC:

14N15 Classical problems, Schubert calculus
57R20 Characteristic classes and numbers in differential topology
57R45 Singularities of differentiable mappings in differential topology
32S20 Global theory of complex singularities; cohomological properties

Keywords:
Thom polynomial; right-left singularity; enumeration of contacts; singular projections

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