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Homology of Hilbert schemes of points on a locally planar curve. (English) Zbl 1409.14011

Let $C$ be an integral proper complex curve with compactified Jacobian $J$. Letting $C^{[n]}$ denote the Hilbert scheme of length $n$ subschemes of $C$, the Abel-Jacobi morphism $\varphi : C^{[n]} \to J$ sends a closed subscheme $Z$ to $\mathcal{I}_Z \otimes \mathcal{O}(x)^{\otimes n}$, where $x \in C$ is a nonsingular point. When $C$ has at most planar singularities, both $C^{[n]}$ and $J$ are integral schemes with local complete intersection singularities according to A. B. Altman et al. [in: Real and compl. Singul., Proc. Nordic Summer Sch., Symp. Math., Oslo 1976, 1–12 (1977; Zbl 0415.14014)] and J. Briancon et al. [Ann. Sci. Éc. Norm. Supér. (4) 14, 1–25 (1981; Zbl 0463.14001)]. Furthermore $\varphi$ has the structure of a $\mathbb{P}^{\infty - g}$-bundle for $n \geq 2g - 1$ by work of A. B. Altman and S. L. Kleiman [Adv. Math. 35, 50–112 (1980; Zbl 0427.14015)] so that the rational homology group $H_*(C^{[n]})$ is determined by $H_*(J)$. Recent work of D. Maulik and Z. Yun [J. Reine Angew. Math. 694, 27–48 (2014; Zbl 1304.14036)] and L. Migliorini and V. Shende [J. Eur. Math. Soc. (JEMS) 15, No. 6, 2353–2367 (2013; Zbl 1303.14019)] endows $H^*(J)$ with a certain perverse filtration $P$ for which $H^*(C^{[n]})$ can be recovered from the $P$-graded space $\text{gr}_P H^*(J)$.

Motivated by these results and a suggestion of Richard Thomas, the author shows how $H_*(C^{[n]})$ can be recovered from a filtration on $H_*(J)$ using a method not reliant on perverse sheaves. Taking an approach inspired by work of H. Nakajima [Ann. Math. (2) 145, No. 2, 379–388 (1997; Zbl 0915.14001)], he defines two pairs of creation and annihilation operators acting on $V(C) = \bigoplus_{n \geq 0} H_*(C^{[n]})$. The first pair $\mu_\pm [pt]$ corresponds to adding or removing a nonsingular point in $C$. The second pair $\mu_\pm[C]$ come from the respective projections $p, q$ from the flag Hilbert scheme $C^{[n,n+1]}$ to $C^{[n]}$ and $C^{[n+1]}$, namely $q,p'$ and $p,q'$ for appropriate Gysin maps $p'$ and $q'$. The main theorem states that the subalgebra of $\text{End}(V(C))$ generated by $\mu_\pm[pt], \mu_{pm}[C]$ is isomorphic to the Weyl algebra $\mathbb{Q}[x_1, x_2, \partial_1, \partial_2]$ and that the natural map $W \otimes \mathbb{Q}[\mu_+[pt], \mu_+[C]] \to V(C)$ is an isomorphism, where $W$ is the intersection of the kernels of $\mu_+[pt]$ and $\mu_-[C]$; moreover the Abel-Jacobi pushforward map $\varphi_* : V(C) \to H_*(J)$ induces an isomorphism $W \cong H_*(J)$. Dual variations for cohomology groups recover and strengthen the results of Maulik-Yun [Zbl 1304.14036] and Migliorini-Shende [Zbl 1303.14019].

Reviewer: Scott Nollet (Fort Worth)

MSC:

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14H20 Singularities of curves, local rings

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