The Gysin map introduced by W. Gysin, originally associated to a map \( f : M \to N \) of closed oriented manifolds, a push-forward covariant homomorphism \( f_* : H^*(M) \to H^*(N) \) between cohomology groups and was later generalized to many different settings. The Gysin homomorphism for fiber bundles and de Rham cohomology of differential forms has a natural interpretation as integration along fibers. The Gysin map was also defined as a push-forward in Chow groups of varieties along proper morphisms of non-singular algebraic varieties.

Gysin maps proved useful in singularity theory and have provided a tool to study the degeneracy loci of morphisms of flag bundles which are related to Schubert manifolds by the Thom-Porteous formula. Most of the results on push-forwards for flag bundles relied on inductive procedures reducing the problem to studying projective bundles. The study of the degeneracy loci of morphisms of flag bundles leads to the development of combinatorial techniques concentrating on the study of Schur polynomials and their generalizations and modifications.

Another direction of study of Gysin homomorphisms was motivated by Quillen’s description of the push-forward maps in complex cobordism using a certain type of a residue, which provided a background for the results of Damon and Akyildiz and Carrell expressing Gysin maps for fiber bundles as Grothendieck residues.

The development of equivariant cohomology had enriched the theory with new tools and a different perspective. Many classical theorems have been rephrased in terms of equivariant characteristic classes. The question of computing Gysin maps for projective bundles can be reduced to studying push-forward maps in equivariant cohomology. A powerful tool to study Gysin maps in equivariant cohomology are localization theorems.

In this paper the author gives a review and an example of computational application of an adaptation in the context of equivariant cohomology of the Pragacz-Ratafiaj formula for push-forwards of Schur classes for Lagrangian Grassmanian manifolds.

For the entire collection see Zbl 1382.14002.

Reviewer: Cenap Özel (Bolu)

MSC:

- 57R91 Equivariant algebraic topology of manifolds
- 14C17 Intersection theory, characteristic classes, intersection multiplicities in algebraic geometry
- 14F43 Other algebro-geometric (co)homologies (e.g., intersection, equivariant, Lawson, Deligne (co)homologies)

Keywords:

Gysin maps; push-forward maps; characteristic classes; equivariant cohomology; Schubert manifolds; flag manifolds; localization; Grothendieck residue; Lagrangian Grassmanians