Banakh, Taras; Bardyla, Serhii; Guran, Igor; Gutik, Oleg; Ravsky, Alex
Positive answers to Koch’s problem in special cases. (English) Zbl 1441.22003
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Summary: A topological semigroup is monothetic provided it contains a dense cyclic subsemigroup. The Koch problem asks whether every locally compact monothetic monoid is compact. This problem was opened for more than sixty years, till Y. Zelenyuk [Fundam. Math. 245, No. 1, 101–107 (2019; Zbl 1418.22001)] obtained a negative answer. In this paper we obtain a positive answer for Koch’s problem for some special classes of topological monoids. Namely, we show that a locally compact monothetic topological monoid $S$ is a compact topological group if and only if $S$ is a submonoid of a quasitopological group if and only if $S$ has open shifts if and only if $S$ is non-viscous in the sense of Averbukh. The last condition means that any neighborhood $U$ of the identity $1$ of $S$ and for any element $a \in S$ there exists a neighborhood $V$ of $a$ such that any element $x \in S$ with $(xV \cup Vx) \cap V \neq \emptyset$ belongs to the neighborhood $U$ of $1$.

MSC:
22A15 Structure of topological semigroups
54D30 Compactness

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Koch’s problem; monothetic semigroup; non-viscous monoid; topological semigroup; semitopological semigroup; cancellative semigroup; locally compact semigroup; countably compact semigroup; feebly compact semigroup; Tkachenko-Tonita group

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