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Summary: A topological vector space over the real or complex field $\mathbb{K}$ is weakly complete if it is isomorphic to a power $\mathbb{K}^J$. For each topological group $G$ there is a weakly complete topological group Hopf algebra $\mathbb{K}[G]$ over $\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$, for which three insights are contributed:

Firstly, there is a comprehensive structure theorem saying that the topological algebra $\mathbb{K}[G]$ is the Cartesian product of its finite dimensional minimal ideals whose structure is clarified.

Secondly, for a compact abelian group $G$ and its character group $\hat{G}$, the weakly complete complex Hopf algebra $\mathbb{C}[G]$ is the product algebra $\mathbb{C}\hat{G}$, with the comultiplication $c : \mathbb{C}\hat{G} \to \mathbb{C}\hat{G} \times \mathbb{C}\hat{G}$, $c(F)(\chi_1, \chi_2) = F(\chi_1 + \chi_2)$ for $F : \hat{G} \to \mathbb{C}$ in $\mathbb{C}\hat{G}$. The subgroup $\Gamma(\mathbb{C}\hat{G})$ of grouplike elements of the group of units of the algebra $\mathbb{C}\hat{G}$ is $\text{Hom}(\hat{G}, (\mathbb{C} \setminus \{0\}))$ while the vector subspace of primitive elements is $\text{Hom}(\hat{G}, (\mathbb{C}, +))$. This forces the group $\Gamma(\mathbb{R}[G]) \subseteq \Gamma(\mathbb{C}[G])$ to be $\text{Hom}(\hat{G}, S^1) \cong \hat{G} \cong G$ with the complex circle group $S^1$. While the relation $\Gamma(\mathbb{C}[G]) \cong G$ remains true for any compact group, $\Gamma(\mathbb{C}[G]) \cong G$ holds for a compact abelian group $G$ if and only if it is profinite.

Thirdly, for each pro-Lie algebra $L$ a weakly complete universal enveloping Hopf algebra $U_K(L)$ over $\mathbb{K}$ exists such that for each connected compact group $G$ the weakly complete real group Hopf algebra $\mathbb{R}[G]$ is a quotient Hopf algebra of $U_K(L(G))$ with the (pro-)Lie algebra $L(G)$ of $G$. The group $\Gamma(U_K(L(G)))$ of grouplike elements of the weakly complete enveloping algebra of $L(G)$ maps onto $\Gamma(\mathbb{R}[G]) \cong G$ and is therefore nontrivial in contrast to the case of the discrete classical enveloping Hopf algebra of an abstract Lie algebra.

MSC:

22E15 General properties and structure of real Lie groups
22E65 Infinite-dimensional Lie groups and their Lie algebras: general properties

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