Wang, Hanfeng; He, Wei
On remainders of locally $s$-spaces. (English) Zbl 1445.54013 Topology Appl. 278, Article ID 107231, 10 p. (2020).

The system ZFC is the set-theoretic framework for this paper. All spaces are assumed to be Tychonoff. If $Y$ is a subspace of a space $X$, then a collection $\mathcal{S}$ of subsets of $X$ is called a source for $Y$ in $X$ if $Y$ is expressible as a union of intersections of non-empty subfamilies of $\mathcal{S}$. A source $\mathcal{S}$ for $Y$ in $X$ is open if all members of $\mathcal{S}$ are open in $X$. A space $X$ is called an $s$-space if there exists a countable open source for $X$ in some (equivalently, in every) Hausdorff compactification of $X$. A space $X$ is a locally $s$-space if every point of $X$ has a neighborhood which is an $s$-space. A space $Y$ is a remainder of a space $X$ if there exists a Hausdorff compactification $bX$ of $X$ such that $Y$ is homeomorphic to $bX \setminus X$.

Being inspired by the results on $s$-spaces shown in [A. V. Arhangel’skii, Commentat. Math. Univ. Carol. 54, No. 2, 121–139 (2013; Zbl 1289.54085)], the authors investigate locally $s$-spaces and their remainders. In particular, the authors show that every metrizable space having at most countably many accumulation points is an $s$-space. Furthermore, a space $X$ with a unique accumulation point is an $s$-space if and only if there exist a metrizable space $Z$ with a unique accumulation point, a compact space $Y$ and a perfect mapping from the direct sum $Y \oplus Z$ onto $X$. The authors prove that if there exists a perfect mapping $f : X \to Y$ of a space $X$ onto a space $Y$, then $X$ is a locally $s$-space if and only if $Y$ is a locally $s$-space. If an $\omega$-narrow semitopological group $G$ is a locally $s$-space, then $G$ is an $s$-space. In consequence, if a countably compact semitopological group $G$ is a locally $s$-space, then $G$ is a compact topological group. A space $X$ is an $s$-space if and only if $X$ is a locally $s$-space such that every (equivalently, some) remainder of $X$ is a locally Lindelöf $\Sigma$-space. The authors show an example of a space $X$ which is not a locally $s$-space but a remainder of $X$ is a locally Lindelöf $\Sigma$-space.

If a locally $s$-space $X$ is not an $s$-space, then no remainder of $X$ is homogeneous. If $X$ is a nowhere locally compact locally $s$-space which has a compactification $bX$ such that $bX \setminus X$ is a locally $s$-space, then both $X$ and $bX \setminus X$ are Lindelöf $p$-spaces. A space $X$ is a locally $s$-space if and only if every (equivalently, some) remainder $Y$ of $X$ is a Lindelöf $\Sigma$-space outside of some compact subspace of $Y$. If a remainder $Y$ of a locally $s$-space $X$ has a $G_\delta$-diagonal, then $Y$ has a countable network and $X$ is an $s$-space. If a remainder $Y$ of a locally $s$-space $X$ is locally perfect, then $X$ is an $s$-space and $Y$ is a Lindelöf $\Sigma$-space. Remainders of locally Lindelöf $\Sigma$-spaces are also investigated. It is shown that every remainder $Y$ of a locally Lindelöf $\Sigma$-space is an $s$-space outside of some compact subspace of $Y$. Some other results and corollaries to the main theorems are included in the article. Finally, the authors show an example of a not locally Lindelöf space $X$ and its remainder $Y$ such that $Y$ is an $s$-space.

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MSC:
54D35 Extensions of spaces (compactifications, supercompactifications, completions, etc.)
54D40 Remainders in general topology
54B05 Subspaces in general topology
54E35 Metric spaces, metrizability
22A05 Structure of general topological groups

Keywords:
s-space; locally $s$-space; Lindelöf $\Sigma$-space; compactification; remainder; source

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References:


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