Akbarov, S. S.
Kernels and cokernels in the category of augmented involutive stereotype algebras. (English)
Zbl 1452.46036

The Pontryagin-van Kampen duality theory can be described as the study of topological abelian groups which can be recovered completely from their one-dimensional representations (characters). In short, a group $G$ is said to satisfy Pontryagin-van Kampen duality ($G$ is reflexive) when the canonical evaluation map of $G$ into its bidual group is a topological isomorphism (here, the dual group $\hat{G}$ is defined as the topological group consisting of all continuous homomorphisms of $G$ into the unit circle of the complex plane, with pointwise product as composition law and equipped with the compact open topology). The application of Pontryagin-van Kampen duality in topological groups has led to many fruitful results. As a consequence, duality theory has now become an essential tool in the study of topological abelian groups.

In the present paper, the author further develops his investigation on the generalization of Pontryagin duality to far more general classes of topological not necessary abelian groups. Here, he suggests a new generalization of Pontryagin duality from the category of abelian locally compact groups to a category which includes all Moore groups, i.e. groups whose irreducible representations are finite-dimensional. Objects in this category are pro-$C^*$-algebras with a structure of Hopf algebras with respect to a certain topological tensor product. The following main statement is proved which is necessary for the generalization of a previous result given by Y. Kaz netsoava [J. Oper. Theory 69, No. 2, 571–600 (2013; Zbl 1299.22005)] to the class of arbitrary Moore groups: a continuous envelope (an analog of the $C^*$-envelope used in [loc. cit.]) $Env_{C^*}(Z \cdot K)$ of the group algebra of a group of the form $Z \cdot K$, where $Z$ is an abelian locally compact group and $K$ is a compact group, is an involutive Hopf algebra in the category of stereotype spaces $(\text{Ste}, \odot)$. This result is a main tool in proving that certain relevant algebras are Hopf algebras in the category AugInvSteAlg of augmented involutive stereotype algebras.

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MSC:

46H05 General theory of topological algebras
46A20 Duality theory for topological vector spaces
46A03 General theory of locally convex spaces
22D15 Group algebras of locally compact groups

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References:
