Define the induced unitary operator by \( U_T : L^2_m(X) \to L^2_m(X) \), \( (U_T f)(x) := f(Tx) \). Let \( T \) be a measure-preserving transformation on some measure space. The automorphism \( T \) is weak mixing if and only if
\[
\lim_{L \to \infty} \frac{1}{L} \sum_{k=0}^{L-1} |(f, U^k_T f)|^2 = 0 \quad \text{for all } f \in [1]^{\perp},
\]
where \( f \in [1]^{\perp} \) is the orthogonal complement of the constant functions in \( L^2_m(X) \) and \( (f, g) = \int_X f g dm \) denotes the inner product in this space.

Let \( G \) be the complete topological group of automorphisms of a Lebesgue space \((X, A, m)\). In this paper, for each \( \alpha \in [0, 1] \), each \( f \in [1]^{\perp} \) and each \( T \in G \), the authors introduce the following:
\[
\mathcal{M}_\alpha(T; f) := \lim \inf_{L \to \infty} \frac{1}{L^\alpha} \sum_{k=0}^{L-1} |(f, U^k_T f)|^2
\]
and
\[
\mathcal{M}^\alpha(T; f) := \lim \sup_{L \to \infty} \frac{1}{L^{1-\alpha}} \sum_{k=0}^{L-1} |(f, U^k_T f)|^2.
\]

The following statement is proved.

Theorem. There exists a generic set \( \mathcal{G} \subseteq G \) of weak-mixing automorphisms such that, for each \( T \in \mathcal{G} \), the set
\[
\{ f \in [1]^{\perp} | \mathcal{M}_\alpha(T; f) = 0 \quad \text{and} \quad \mathcal{M}^\alpha(T; f) = \infty, \forall 0 < \alpha < 1 \}
\]
is generic in \([1]^{\perp}\).

Then, the authors present generic sets of measures with respect to topological shifts over finite alphabets and Axiom A diffeomorphisms over topologically mixing basic sets.

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MSC:

37A05 Dynamical aspects of measure-preserving transformations
37A10 Dynamical systems involving one-parameter continuous families of measure-preserving transformations
37A25 Ergodicity, mixing, rates of mixing
28D05 Measure-preserving transformations

Keywords:

scales of mixing; Koopman operator; Halmos lemma; Rohlin lemma

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References:


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