In recent years, many of the results of the Minimal Model Program have been extended from characteristic zero to characteristic $p > 5$. For example, existence of log minimal models of threefolds has been proved by C. Birkar [Ann. Sci. Éc. Norm. Supér. (4) 49, No. 1, 169–212 (2016; Zbl 1346.14040)], C. D. Hacon and C. Xu [J. Am. Math. Soc. 28, No. 3, 711–744 (2015; Zbl 1326.14032)]. There has also been significant progress on the log abundance conjecture. In the case of threefold klt pairs, it has been proved when the variety is of log general type or when the boundary divisor is big (in addition to the above references, see also [P. Cascini et al., Ann. Sci. Éc. Norm. Supér. (4) 48, No. 5, 1239–1272 (2015; Zbl 1408.14020]) and [C. Xu, J. Inst. Math. Jussieu 14, No. 3, 577–588 (2015; Zbl 1346.14020)]).

In the present paper, the author builds upon his previous work [L. Zhang, J. Lond. Math. Soc., II. Ser. 99, No. 2, 332–348 (2019; Zbl 1410.14013)] and proves abundance for threefolds with non-trivial Albanese map.

**Theorem 1.1.** Let $X$ be a klt, $\mathbb{Q}$-factorial, projective minimal threefold defined over an algebraically closed fields $k$ of characteristic $p > 5$. Assume that the Albanese map is non-trivial. Then $K_X$ is semi-ample.

The author also proves some instances of log abundance.

**Theorem 1.2.** Let $(X, B)$ be a klt, $\mathbb{Q}$-factorial, projective minimal pair of dimension three defined over an algebraically closed field of characteristic $p > 5$. Assume that the Albanese map $\alpha_X$ is non-trivial. Denote by $f : X \to Y$ the fibration arising from the Stein factorization of $\alpha_X$ and by $X_\eta$ the generic fiber of $f$. Assume moreover that $B = 0$ if

1. $\dim(Y) = 2$ and $\kappa(X_\eta, (K_X + B)|_{X_\eta}) = 0$, or
2. $\dim(Y) = 1$ and $\kappa(X_\eta, (K_X + B)|_{X_\eta}) = 1$.

Then $K_X + B$ is semi-ample.

Reviewer: Justin Lacini (Lawrence)
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