Peng, Liang-Xue; Liu, Ying
On Lindelöf feathered topological groups. (English) Zbl 1457.54031

M. G. Tkachenko [Topology Appl. 38, No. 1, 21–37 (1991; Zbl 0722.54039)] introduced the notion of $R$-factorizable topological groups: A topological group $G$ is $R$-factorizable if for any continuous real-valued function $f$ on $G$ there exist a continuous homomorphism $p : G \to H$ onto a second-countable topological group $H$ and a continuous real-valued function $g$ on $H$ such that $f = p \circ g$. One of the first factorization theorems for topological groups states that any compact topological group is $R$-factorizable.

In this paper, the authors define the notion of $PR$-factorizable topological groups and investigate some properties of $PR$-factorizable topological groups. A topological group $G$ is called $PR$-factorizable if for every continuous real-valued function $f$ on $G$ there exist a perfect homomorphism $p : G \to H$ onto a second-countable topological group $H$ and a continuous real-valued function $g$ on $H$ such that $f = p \circ g$. From the definition, every $PR$-factorizable topological group is $R$-factorizable.

The authors prove that for any topological group $G$ the following are equivalent: (1) $G$ is $PR$-factorizable; (2) $G$ is Lindelöf feathered; (3) $G$ is $\omega$-narrow and feathered; (4) $G$ is a Lindelöf $p$-space; (5) there exists a compact invariant subgroup $H$ of $G$ such that the quotient group $G/H$ is separable metrizable.

As a result, it is shown that the product of countably many $PR$-factorizable topological groups, any closed subgroup of a $PR$-factorizable topological group, and any image under an open continuous homomorphism of a $PR$-factorizable topological group are all $PR$-factorizable. The authors also discuss some kinds of completeness type properties in the class of $PR$-factorizable groups. Finally, they give an example of an $R$-factorizable topological group that is not $PR$-factorizable.

Reviewer: Kohzo Yamada (Shizuoka)

MSC:
54H11 Topological groups (topological aspects)
54B20 Hyperspaces in general topology
54E35 Metric spaces, metrizability

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Lindelöf feathered; $PR$-factorizable; Čech-complete; Sánchez-Okunev complete; topological group

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References:

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