Given a smooth closed oriented manifold $M$ and an exotic sphere $\Sigma$ of the same dimension, one can form the connected sum $M \# \Sigma$. Sometimes this yields a manifold diffeomorphic to $M$, if $\Sigma$ lies in the inertia group of $M$, but often $M \# \Sigma$ is not diffeomorphic to $M$. In the latter case, it is an interesting question to what extent the cohomology $H^\ast(B\text{Diff}^+(M); \mathbb{Z})$ of the classifying space of the topological group of orientation-preserving diffeomorphisms of $M$ differs from $H^\ast(B\text{Diff}^+(M \# \Sigma); \mathbb{Z})$.

In this paper, it is proven when $M$ is simply-connected and of even dimension $2n \geq 6$, then in a range these cohomology groups are isomorphic after replacing the coefficients $\mathbb{Z}$ by $\mathbb{Z}[\frac{1}{k}]$ with $k$ the order of $\Sigma$ in the finite group $\Theta_{2n}$ of oriented exotic $2n$-spheres. This range depends on the “stable genus” of $M$, in the sense of S. Galatius and O. Randal-Williams [J. Am. Math. Soc. 31, No. 1, 215–264 (2018; Zbl 1395.57044); Ann. Math. (2) 186, No. 1, 127–204 (2017; Zbl 1412.57026); Acta Math. 212, No. 2, 257–377 (2014; Zbl 1377.55012)]. Furthermore, infinitely many families of examples are constructed to illustrate that it is necessary to invert the order of $\Sigma$.

The main input is the aforementioned work of Galatius and Randal-Williams. This distinguishes it from earlier work of W. G. Dwyer and R. H. Szczarba [Ill. J. Math. 27, 578–596 (1983; Zbl 0507.57016)], which uses smoothing theory instead to prove a similar result about the classifying space of the identity component of $\text{Diff}^+(M)$.

Reviewer: Alexander Kupers (Toronto)

MSC:

55R40 Homology of classifying spaces and characteristic classes in algebraic topology
57R60 Homotopy spheres, Poincaré conjecture
57S05 Topological properties of groups of homeomorphisms or diffeomorphisms

Keywords:

manifolds; diffeomorphisms; characteristic classes

Full Text: DOI

References:


[22] Hovey, MA; Ravenel, DC, The \(\langle 7\rangle\)-connected cobordism ring at \(\langle p=3\rangle\), Trans. Am. Math. Soc., 347, 9, 3473-3502 (1995) - Zbl 0852.55008


This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.