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Remarks on $g$-reversible topological groups. (English) [Zbl 1461.22001]

All topological groups in the paper under this review are Hausdorff. Chatyrko and Shakhmatov introduced the notion of $g$-reversible topological group and the notion of hereditarily $g$-reversible topological group [V. Chatyrko et al., Topology Appl. 275, Article ID 107051, 18 p. (2020; Zbl 1437.22001)]. A topological group $G$ is called $g$-reversible if each continuous automorphism of $G$ is open. A topological group with all subgroups $g$-reversible is called hereditarily $g$-reversible. The authors of the paper under review give the full solution of some problems which were formulated by Chatyrko and Shakhmatov. It is proved that a $g$-reversible locally compact abelian group may have a non-$g$-reversible closed subgroup. The authors prove that if $D$ is a discrete group with $|D| < \varepsilon$ then for any compact group $K$, $D \times K$ is $g$-reversible. It is proved that for every non-trivial topological abelian group $G$, the product group $G^\omega$ is not hereditarily $g$-reversible. The authors give the full description of compact connected abelian groups and compact totally disconnected abelian groups which are hereditarily $g$-reversible. For example, a compact connected abelian group $G$ is hereditarily $g$-reversible if and only if $G \cong \hat{\mathbb{Q}}^n$ where $n$ is a natural number.

Finally they give an algebraic description of abelian groups which admit hereditarily $g$-reversible compact topological group topologies. An abelian group $G$ admits hereditarily $g$-reversible compact group topology if and only if $G \cong \mathbb{R}^m \times \prod_{p \in \mathcal{P}} (\mathbb{Z}_p^m \times F_p)$ where $m = 0$ or $1$, $m_p$ is a non-negative integer and $F_p$ is a finite $p$-group.

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22D05 General properties and structure of locally compact groups
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References: