Hacon, Christopher D.; McKernan, James; Xu, Chenyang

Boundedness of moduli of varieties of general type. (English) Zbl 1464.14038

The moduli functor (see Definition 1.5, the case with no boundary) of semi log canonical (slc) models with prescribed Hilbert function $H$ is assigning to any $S$ the flat projective morphisms $X \to S$ whose fibers are slc models with ample canonical class and Hilbert function $H$, the canonical $\omega_X$ is flat over $S$ and all reflexive powers of $\omega_X$ commute with base change. The main result of this paper (see Theorem 1.1) is showing that the moduli functor is bounded, getting a bounded family when fixing the degree. To be precise: for $n$ integer, $d$ positive rational number and $I \subset [0,1]$ satisfying the descending chain condition, the set $F(n, d, I)$ of log pairs $(X, \Delta)$ such that (i) $X$ is projective of dimension $n$, (ii) $(X, \Delta)$ is slc, (iii) the coefficients of $\Delta$ belong to $I$, (iv) $K_X + \Delta$ is an ample $\mathbb{Q}$-divisor, and (v) $(K_X + \Delta)^n = d$, is bounded, which in particular means the existence of finite set $I_0$ such that $F(n, d, I) = F(n, d, I_0)$. The proof can be reduced to the case of irreducible log canonical pairs (via normalization and Theorem 1.6, see details in the Introduction of the paper under review) and then it follows from an abundance theorem for families (see Theorem 1.2): $(X, \Delta)$ a log pair, the coefficients of $\Delta$ in $(0,1] \cap \mathbb{Q}$, $\pi : X \to U$ projective morphism to smooth $U$ such that $(X, \Delta)$ is log smooth over $U$. If there is a closed point in $U$ whose fiber has a good minimal model then every fiber has a good minimal model and also $(X, \Delta)$ has a good minimal model over $U$.

Reviewer: Roberto Muñoz (Madrid)

MSC:
14J10 Families, moduli, classification: algebraic theory
14E30 Minimal model program (Mori theory, extremal rays)

Keywords:
moduli; boundedness; general type; minimal model program; abundance

Full Text: DOI