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A countable dense homogeneous topological vector space is a Baire space. (English)


In this article the authors study the notion of a space being countable dense homogeneous (CDH) and the meaning of Baireness. All the results of the paper are very interesting, providing new and significant facts on related topics. Especially, some of the main results state that:

- Every homogeneous CDH space $X$ containing a copy of the Cantor set is a Baire space.
- Every CDH topological group $G$ is a Baire space.
- Every CDH topological vector space $E$ is a Baire space.

Moreover, the authors prove that:

- If $E$ is an infinite-dimensional normed space and $F$ is a linear subspace of the dual $E^*$, separating points of $E$, then the space $E$ equipped with the weak topology generated by $F$ is not CDH.
- For a separable space $X$ (not necessarily metrizable) and an infinite cardinal $\kappa$, if $X^\kappa$ is CDH, then $X$ and $X^\kappa$ are Baire spaces.
- If $X$ and $Y$ are two crowded spaces and $X \times Y$ is CDH, then $X$ contains a copy of the Cantor set if and only if $Y$ contains a copy of the Cantor set.

Also, they investigate the relation of CDH with the space $C_p(X)$ of all continuous real-valued functions on a space $X$, endowed with the pointwise topology. Some of the main results are given as follows:

- If $X$ is a space containing an infinite bounded subset $A$, then $C_p(X)$ is not CDH.
- If $X$ is a metrizable space and $C_p(X)$ is CDH, then $X$ is discrete.
- If $X$ is an uncountable separable metrizable space, then $C_p(X)$ is not CDH.

Finally, the authors provide examples of hereditary Baire separable pre-Hilbert spaces which are not CDH.

Reviewer: Dimitrios Georgiou (Patras)

MSC:

54C35 Function spaces in general topology
54E52 Baire category, Baire spaces
46A03 General theory of locally convex spaces
22A05 Structure of general topological groups

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function space; pointwise convergence topology; $C_p(X)$ space; countable dense homogeneity; Baire space; topological vector space; property (B)

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References:
