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The homotopy types of $PSp(n)$-gauge groups over $S^{2m}$. (English) Zbl 1465.55001


Given a topological group $G$, the isomorphism classes of principal $G$-bundles over $S^n$ are classified by elements in $\pi_{n-1}(G)$. In particular, when (1) $G = PSp(2)$ and $n = 8$ or (2) $G = PSp(3)$ and $n = 4$, $\pi_{n-1}(G) \cong \mathbb{Z}$. Therefore any principal $G$-bundle $P$ is classified by an integer $k$ in these two cases. The gauge group of $P$ is the topological group consisting of $G$-equivariant automorphisms of $P$ that fix $S^n$, and is denoted by $G_k(S^n, G)$. In this paper the author shows the following theorem:

**Theorem:** Let $(k, l)$ be the greatest common divisor of the integers $k$ and $l$.

- If $G_k(PSp(2), S^8) \cong G_l(PSp(2), S^8)$, then $(140, k) = (140, l)$;
- if $(140, k) = (140, l)$, then $\Omega G_k(PSp(2), S^8) \cong \Omega G_l(PSp(2), S^8)$;
- if $G_k(PSp(3), S^4) \cong G_l(PSp(3), S^4)$, then $(84, k) = (84, l)$;
- if $(672, k) = (672, l)$, then $\Omega G_k(PSp(3), S^4) \cong \Omega G_l(PSp(3), S^4)$ after localization at any prime.

In Section 2 the author gives the background of his method. According to [M. F. Atiyah and R. Bott, Philos. Trans. R. Soc. Lond., Ser. A 308, 523–615 (1983; Zbl 0509.14014); D. H. Gottlieb, Trans. Am. Math. Soc. 171, 23–50 (1972; Zbl 0251.55018)], the classifying space of the gauge group $BG_k(S^n, G)$ is homotopy equivalent to the connected component of $\text{Map}(S^n, BG)$ that contains $k\epsilon$. Consider the homotopy fibration sequence

$$G \overset{\alpha_k}{\longrightarrow} \Omega^{n-1}G \longrightarrow BG_k \overset{ev}{\longrightarrow} BG,$$

where $\alpha_k$ is the evaluation map at the base point, $\Omega^{n-1}G$ is the connected component of $\Omega G$ that contains the identity and $\alpha_k$ is a connecting map. The adjoint of $\alpha_k$ is the Smaslen product $k(\epsilon, id_G) : S^{n-1} \land G \rightarrow G$, where $\epsilon : S^n \rightarrow BG$ is a map representing the generator of $\pi_n(BG) \cong \mathbb{Z}$ and $id_G$ is the identity map on $G$. We define the order of $(\epsilon, id)$ to be the minimum positive integer $m$ such that $m(\epsilon, id)$ is null homotopic. It is known that the greatest common divisor $(k, m)$ can essentially determine the homotopy type of $G_k(S^n, G)$. In Section 3 the author calculates the order $m$ of $(\epsilon, id_G)$ for $G_k(PSp(2), S^8)$ and in Section 4 calculates $m$ for $G_k(PSp(3), S^4)$.

In the $PSp(2)$ case, let $\epsilon : S^7 \rightarrow PSp(2)$ and $\tau : S^7 \rightarrow Sp(2)$ be maps representing the generators of $\pi_7(PSp(3))$ and $\pi_7(Sp(3))$. Using the fibration sequence

$$Sp(2) \rightarrow PSp(2) \rightarrow B\mathbb{Z}/2\mathbb{Z},$$

the author shows that the orders of $(\tau, id_{Sp(2)})$ and $(\epsilon, id_{PSp(2)})$ are the same. By [H. Hamanaka et al., Topology Appl. 155, No. 11, 1207–1212 (2008; Zbl 1144.55014)] the order of $(\tau, id_{Sp(2)})$ is 140, so the author obtains the first two statements of his theorem. In the $PSp(3)$ case, the author uses the same method as [T. Cutler, Topology Appl. 236, 44–58 (2018; Zbl 1383.55005)] to obtain the last two statements of the theorem.

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