In the paper under review, the authors prove that if $X \subset P^n$ is a smooth projective variety, then the derived categories of coherent sheaves $D(X \times P^n T)$ and $D(X^3 \times P^n T^3)$ admit semiorthogonal decompositions with isomorphic primitive parts. The primitive part is the component not coming from the ambient space. Kuznetsov’s paper considered only the case where $T$ is a linear subspace.

The first part of the article is a nice introduction to decomposition of derived categories and to HP theory. After the statement of the main result, the authors show some applications: counterexamples to the birational Torelli conjecture for Calabi-Yau threefolds and fivefolds, and a relation between the Euler characteristics of $X, T$, and their HP-duals.

The proof of the main result is long and technical, and expands Kuznetsov’s original approach, which is well explained in [R. P. Thomas, Proc. Symp. Pure Math. 97, 585–609 (2018; Zbl 1451.14052)].

Let $H \subset X \times T^3$ be defined by the incidence relation. The authors consider the embeddings $I_1 : D(X \times P^n T) \hookrightarrow D(H)$ and $I_2 : D(X^3 \times P^n T^3) \hookrightarrow D(H)$. They show that the pull-back $I_1^* : D(H) \to D(X \times P^n T)$ is fully faithful on some subcategories coming from the Lefschetz decompositions of $D(X)$ and $D(T)$. The same result holds for $I_2^* : D(H) \to D(X^3 \times P^n T^3)$, with the appropriate change of notation. The Lefschetz decompositions are not assumed to be rectangular. Finally, they prove that $I_2^* I_1$ gives the isomorphism between the principal parts of $D(X \times P^n T)$ and $D(X^3 \times P^n T^3)$.

The underlying idea of the proof is to consider a decomposition of the subcategory $D(X \times P^n T)^\perp \subset D(H)$, where the pieces are nicely ordered in a “chess board”, and study the relations between each piece and the others pieces of the board.

Reviewer: Giosuè Muratore (Roma)

MSC:

14F08 Derived categories of sheaves, dg categories, and related constructions in algebraic geometry
18G80 Derived categories, triangulated categories

Keywords:
categorification; Plücker formula; homological projective duality

Full Text: DOI

References:


This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.