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On feebly compact paratopological groups. (English) Zbl 1468.22003

Summary: We obtain many results and solve some problems about feebly compact paratopological groups. We obtain necessary and sufficient conditions for such a group to be topological. One of them is the quasiregularity. We prove that each 2-pseudocompact paratopological group is feebly compact and that each Hausdorff σ-compact feebly compact paratopological group is a compact topological group. Our particular attention concerns periodic and topologically periodic groups. We construct examples of various compact-like paratopological groups which are not topological groups, among them a $T_0$ sequentially compact group, a $T_1$-2-pseudocompact group, a functionally Hausdorff countably compact group (under the axiomatic assumption that there is an infinite torsion-free Abelian countably compact topological group without non-trivial convergent sequences), and a functionally Hausdorff second countable group sequentially precompact group. We prove that the product of a family of feebly compact paratopological groups is feebly compact, and that a paratopological group $G$ is feebly compact provided it has a feebly compact normal subgroup $H$ such that a quotient group $G/H$ is feebly compact. For our research we also study some general constructions of paratopological groups. We extend the well-known construction of Raĭkov completion of a $T_0$ topological group to the class of paratopological groups. We investigate cone topologies of paratopological groups which provide a general tool for constructing pathological examples, especially examples of compact-like paratopological groups with discontinuous inversion. We find a simple interplay between the algebraic properties of a basic cone subsemigroup $S$ of a group $G$ and compact-like properties of two basic semigroup topologies generated by $S$ on the group $G$.

MSC:
22A15 Structure of topological semigroups
54D10 Lower separation axioms ($T_0$–$T_3$, etc.)
54H11 Topological groups (topological aspects)
54H99 Connections of general topology with other structures, applications

Keywords:
paratopological group; continuity of the inverse; totally countably compact paratopological group; countably compact paratopological group; 2-pseudocompact paratopological group; saturated paratopological group; topologically periodic paratopological group; product of paratopological groups; pseudocompact topological group; countably compact topological group; countably pracompact space

Full Text: DOI

References:
[11] Banakh, T.; Dimitrova, S.; Gutik, O., Embedding the bicyclic semigroup into countably compact topological semigroups,


[34] Engelking, R., General Topology (1989), Heldermann: Heldermann Berlin - Zbl 0684.54001


[44] Hart, K. P.; van Mill, J., A countably compact H such that \( \mathbb{H} \setminus \text{times H} \) is not countably compact, Trans. Am. Math. Soc., 323, 811-821 (Feb 1991)


[53] Lin, F.; Lin, S., Pseudobounded or $\omega$-pseudobounded paratopological groups, Filomat, 25, 3, 93-103 (2011) - Zbl 1265.54133


[59] Lippipari, P., A very general covering property - Zbl 1265.54104


[62] Matveev, M., A survey on star covering properties


[69] Raven, A., The topological and algebraical properties of paratopological groups (2002), Lviv University, (in Ukrainian)

[70] A. Ravsky, The continuity of inverse in groups: the survey and new results, in: I-st Summer School in Topological Algebra


[73] Raven, A., Post #209491 at MathOverflow

[74] Raven, A., Čech complete semitopological groups are topological groups, preprint.


[76] E. Reznichenko, Čech complete semitopological groups are topological groups, preprint.


[78] E. Reznichenko, Čech complete semitopological groups are topological groups, preprint.


[80] Romaguera, S.; Sanchis, M., Continuity of the inverse in pseudocompact paratopological groups, Algebra Colloq., 14, 1, 167-175 (2007) - Zbl 1110.22001


[86] Stephenson, R. M., Initially $\kappa$-compact and related compact spaces, (Kunen, K.; Vaughan, J. E., Handbook of Set-Theoretic

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Topology (1984), Elsevier), 603-632 · Zbl 0588.54025


Tkachenko, M., Countably compact and pseudocompact topologies on free Abelian groups, Soviet Math. (Iz. VUZ), 34, 5, 79-86 (1990) · Zbl 0714.22001

Tkachenko, M., Semitopological and paratopological groups vs topological groups, (Hart, K. P.; van Mill, J.; Simon, P., Recent Progress in General Topology III (2013)), 803-859

Tkachenko, M., Productive properties in topological groups, preprint (version April 18, 2013)


Vaughan, J. E., Countably compact and sequentially compact spaces, (Kunen, K.; Vaughan, J. E., Handbook of Set-Theoretic Topology (1984), Elsevier), 569-602 · Zbl 0562.54031


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