Freixas i Montplet, Gerard; von Pippich, Anna-Maria

This work is a generalization to Riemann surfaces $X_\Gamma = \Gamma \backslash \mathbb{H}^2$ of a Grothendieck-Riemann-Roch formula established by P. Deligne [Contemp. Math. 67, 93–117 (1987; Zbl 0629.14008)] and J.-M. Bismut, H. Gillet and C. Soulé [Commun. Math. Phys. 115, No. 1, 49–78, 79–126; No. 2, 301–351 (1988; Zbl 0651.32017)]. Here, $\Gamma$ is a Fuchsian group of the first kind with singularities of cusp or orbifold type (corresponding to parabolic and elliptic elements of $\Gamma$). The main theorem relates the determinant of some cohomology space with a Quillen type metric, Selberg zeta function values and geometry of bundles on moduli spaces of pointed orbicurves. The novelty of this important work is to permit singularities on the surface $X_\Gamma$: the results of this paper apply to the modular group $\Gamma = \text{PSL}_2(\mathbb{Z})$ in particular. The core of the paper consists of deep analytic surgery techniques through Mayer-Vietoris formulas for determinants of Laplacians introduced by D. Burghelea, L. Friedlander and L. Kappeler [J. Funct. Anal. 107, No. 1, 34–65 (1992; Zbl 0759.58043)]. With appropriate domains obtained by taking off small $\varepsilon$-neighborhood around every singularity and performing the spectral analysis for the $\varepsilon \to 0$ limit, the authors reduce the determinants for the Laplace-Beltrami or Dirichlet-to-Neumann operators to explicit expressions for model hyperbolic cusps and cones. Such computations, which appear notably in theoretical physics literature, are developed here with full mathematical rigor; they can be used for several variants in the geometry of bundles on moduli spaces, such as the Burgos-Kramer-Kühn program of extending arithmetic intersection theory to singular Hermitian vector bundles (see U. Kühn [J. Reine Angew. Math. 534, 209–236 (2001; Zbl 1084.14028)]). As consequences of the main theorem, let us quote the proof of an arithmetic Riemann-Roch formula in the realm of Arakelov geometry and an expression of the Selberg zeta special value $Z'(1, \text{PSL}_2(\mathbb{Z}))$ in terms of logarithmic derivatives of Dirichlet L-function values.

Reviewer: Laurent Guillopé (Nantes)

MSC:

14C40 Riemann-Roch theorems
14G40 Arithmetic varieties and schemes; Arakelov theory; heights
11F72 Spectral theory; trace formulas (e.g., that of Selberg)
58J52 Determinants and determinant bundles, analytic torsion
14F43 Other algebro-geometric (co)homologies (e.g., intersection, equivariant, Lawson, Deligne (co)homologies)

Keywords:
Riemann-Roch theorem; Arakelov geometry; Fuchsian group; Quillen metric; analytic surgery; Mayer-Vietoris; Dirichlet-to-Neumann; Selberg zeta function; L-functions

Full Text: DOI

References:
