Codimension one distributions of degree 2 on the three-dimensional projective space. (English) [Zbl 1470.14035]

A codimension one distribution $D$ on $\mathbb{P}^n$ is given by a short exact sequence

$$0 \to T_D \to T_{\mathbb{P}^n} \xrightarrow{\omega} \mathcal{I}_Z(d+2) \to 0$$

where the corresponding singular scheme $Z = \text{Sing}(D)$ has codimension at least two and $\omega$ is given dually by a twisted 1-form $\omega \in H^0(\Omega^1_{\mathbb{P}^n}(d+2))$. The tangent sheaf $T_D = \ker \omega$ of $D$ is reflexive of rank $n-1$ and $d$ is the degree of $D$. O. Calvo-Andrade et al. developed a classification of codimension one distributions in terms of Chern classes of the tangent sheaf $T_D$ [Int. Math. Res. Not. 23, 9011–9074 (2020; Zbl 1460.32040)] and classified the distributions of degrees $d = 0$ or $d = 1$ on $\mathbb{P}^n$ and those of degree $d = 2$ on $\mathbb{P}^3$ with $T_D$ locally free and $Z$ reduced. Here the authors complete the classification of codimension one distributions of degree $d = 2$ on $\mathbb{P}^3$ without restrictions. Their main result describes all such tangent sheaves $T_D$ in terms of Chern numbers $c_2, c_3$ ($d = 2$ is equivalent to $c_1 = 0$), stability, the spectrum of $T_D$, and the purely 1-dimensional part $\text{Sing}_1(D)$ of $Z$. The cases where $c_2(T_D) \leq 1$ were handled by Calvo-Andrade, Corrêa and Jardim [loc. cit.]. On the other hand, when $c_2(T_D) \geq 2$ the authors determine the invariants of the locally Cohen-Macaulay curve $\text{Sing}_1(D)$ in terms of the Chern numbers $c_2(T_D), c_3(T_D)$: the degree of $\text{Sing}_1(D)$ is at most four, but such Cohen-Macaulay curves were classified by S. Nollet [Ann. Sci. Éc. Norm. Supér. 30, 367–384 (1997; Zbl 0892.14004)] and S. Nollet and E. Schlesinger [Compos. Math. 139, 169–196 (2003; Zbl 1053.14035)]. The authors use a variety of techniques to construct the corresponding distributions.

Reviewer: Scott Nollet (Fort Worth)

MSC:

14F06 Sheaves in algebraic geometry
32M25 Complex vector fields, holomorphic foliations, $\mathbb{C}$-actions
32S65 Singularities of holomorphic vector fields and foliations
14D20 Algebraic moduli problems, moduli of vector bundles
14J60 Vector bundles on surfaces and higher-dimensional varieties, and their moduli
14D22 Fine and coarse moduli spaces
13D02 Syzygies, resolutions, complexes and commutative rings

Keywords:

holomorphic distributions; sheaves; singular scheme

Software:

Macaulay2

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References:
