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Quartic monoid surfaces with maximum number of lines. (English) Zbl 1470.14070

Summary: In 1884 the German mathematician K. Rohn published a substantial paper [Math. Ann. 24, 55–152 (1883; JFM 16.0705.01)] on the properties of quartic surfaces with triple points, proving (among many other things) that the maximum number of lines contained in a quartic monoid surface is 31. In this paper we study in details this class of surfaces. We prove that there exists an open subset $A \subseteq \mathbb{P}^1_K (K$ is a characteristic zero field) that parametrizes (up to a projectivity) all the quartic monoid surfaces with 31 lines; then we study the action of $\text{PGL}(4, K)$ on these surfaces, we show that the stabiliser of each of them is a group isomorphic to $S_3$ except for one surface of the family, whose stabiliser is a group isomorphic to $S_3 \times C_3$. Finally, given two quartic surfaces $Q(a)$ and $Q(b)$, with $a, b \in A$, we show that $Q(a)$ and $Q(b)$ are projectively equivalent if and only if $j(a) = j(b)$, where $j$ is the $j$-function.

To get our results, several computational tools, available in computer algebra systems, are used.

MSC:
14J28 $K3$ surfaces and Enriques surfaces
14J26 Rational and ruled surfaces
14J17 Singularities of surfaces or higher-dimensional varieties
14Q10 Computational aspects of algebraic surfaces

Keywords:
quartic monoid surface; algebraic surface; incidence structure; constructive geometry; $j$-function

Software:
CoCoA; SageMath

Full Text: DOI

References:
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[7] Harris, J., Algebraic Geometry, a First Course (1992), Springer-Verlag · Zbl 0779.14001

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