The article under review treats rack objects in a general category. Here a rack (in the category of sets) is a set $R$ with two binary operations $\triangleright, \triangleleft : R \times R \to R$ satisfying the following axioms for all $a, b, c$ in $R$:

(i) $a \triangleright (b \triangleright c) = (a \triangleright b) \triangleright (a \triangleright c)$
(ii) $(a \triangleleft b) \triangleleft c = (a \triangleleft c) \triangleleft (b \triangleleft c)$
(iii) $(a \triangleright b) \triangleleft a = a \triangleright (b \triangleleft a) = b$.

The author extends this definition to racks in any category $C$ with finite products. This definition is different from the one adopted in [J. S. Carter et al., J. Homotopy Relat. Struct. 3, No. 1, 13–63 (2008; Zbl 1193.18009)] where the authors consider objects with a given diagonal (leading then to racks in coalgebras, close to rack bialgebras). The study in the present article is also different from [A. S. Crans and F. Wagemann, Homology Homotopy Appl. 16, No. 2, 85–106 (2014; Zbl 1350.18004)] where the authors consider category objects in the category of racks, while the present article studies rack objects in the categories of racks, groups and abelian groups.

The upshot of this study are no-go theorems, like for example the statement that a simple group admits only quandles and permutation racks (Cor. 4.4). This leads then to a classification (Thm. 5.2) of the various categories of racks in abelian groups, showing that these are isomorphic to module categories over specified commutative rings.

The last part of the article uses this study to exclude certain topological rack structures on specific topological spaces $X$ by means of the (standard) homotopy functors $\Pi_n(X)$. For example, Theorem 6.1 states that for a path-connected topological space $R$ such that there exists an $n$ such that $\Pi_n(R)$ does not admit a symmetric quandle-in-abelian-groups structure, the $R$ does not admit a topological symmetric quandle-in-abelian-groups structure. The article closes showing that any rack-in-abelian-groups structure on a topological group $G$ can be realized as homotopy rack of the classifying space of $G$.

Reviewer: Friedrich Wagemann (Nantes)

MSC:
18C40 Structured objects in a category (group objects, etc.)
16B50 Category-theoretic methods and results in associative algebras (except as in 16D90)
20J15 Category of groups
57K10 Knot theory
57K12 Generalized knots (virtual knots, welded knots, quandles, etc.)

Keywords: category theory; rack theory; categorical racks

Full Text: DOI

References:


