Summary: We show that for an elliptic curve $E$ defined over a number field $K$, the group $E(\mathbb{A}_K)$ of points of $E$ over the adele ring $\mathbb{A}_K$ of $K$ is a topological group that can be analyzed in terms of the Galois representation associated to the torsion points of $E$. An explicit description of $E(\mathbb{A}_K)$ is given, and we prove that for $K$ of degree $n$, ‘almost all’ elliptic curves over $K$ have an adelic point group topologically isomorphic to 

$$(\mathbb{R}/\mathbb{Z})^n \times \hat{\mathbb{Z}}^n \times \prod_{m=1}^{\infty} \mathbb{Z}/m\mathbb{Z}.$$ 

We also show that there exist infinitely many elliptic curves over $K$ having a different adelic point group.

MSC:

11G05 Elliptic curves over global fields
11G07 Elliptic curves over local fields
11F80 Galois representations

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elliptic curves; adelic points; Galois representation

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