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The homotopy types of \(U(n)\)-gauge groups over lens spaces. (English) [Zbl 1473.55006]


For coprime integers \(p\) and \(q\), the lens space \(L(p,q)\) is the quotient of \(S^3\) by the \(\mathbb{Z}/p\mathbb{Z}\)-action \((z_0, z_1) \mapsto (e^{2\pi i/p} z_0, e^{2\pi q/p} z_1)\). It admits a CW-structure \(P^2(p) \cup e^3\) where \(P^2(p)\) is the mapping cone of the degree map \(p : S^1 \to S^1\).

Let \(P\) be a principal \(U(n)\)-bundle over \(L(p,q)\) where \(p\) is a prime. The isomorphism class of \(P\) is determined by its first Chern class \(\ell \in H^2(L(p,q); \mathbb{Z})\) which is \(\mathbb{Z}/p\mathbb{Z}\). The gauge group of \(P\), denoted by \(G_k(L(p,q))\), is the topological group consisting of \(U(n)\)-equivariant automorphisms of \(P\) that fix \(L(p,q)\). It is known that gauge groups of \(U(n)\)-bundles over \(L(p,q)\) have finitely many distinct homotopy types. Classifying the homotopy types of various \(G_k(L(p,q))\) is important to understand the topology of these gauge groups.

In Section 2 the authors study the homotopy theory of \(L(p,q)\) and show that the inclusion \(P^2(p) \hookrightarrow L(p,q)\) induces an isomorphism between \([L(p,q), BU(n)]\) and \([P^2(p), BU(n)]\) which are \(\mathbb{Z}/p\mathbb{Z}\).

Denote by \(G_k(P^2(p))\) the gauge group of the principal \(U(n)\)-bundle over \(P^2(p)\) with first Chern class \(k \in H^2(P^2(p); \mathbb{Z}) \cong \mathbb{Z}/p\mathbb{Z}\). In Section 3 the authors classify the homotopy types of various \(G_k(P^2(p))\) and show that \(G_k(P^2(p)) \simeq G_\ell(P^2(p))\) if and only if \(gcd(p,k) = gcd(p,\ell)\).

Combining their work in Sections 2 and 3, in Section 4 the authors prove that \(gcd(p,k) = gcd(p,\ell)\) if \(G_k(L(p,q)) \simeq G_\ell(L(p,q))\). Moreover, they show the converse whenever there exists an integer \(u\) such that \(k \equiv u\ell \pmod{p}\) and \(u^2 \equiv \pm 1 \pmod{p}\). In particular, for \(p \in \{3, 5\}\), \(G_k(L(p,q)) \simeq G_\ell(L(p,q))\) if and only if \(gcd(p,k) = gcd(p,\ell)\).

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MSC:

- 55P15 Classification of homotopy type
- 54C35 Function spaces in general topology
- 81T13 Yang-Mills and other gauge theories in quantum field theory

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- lens space; gauge group; homotopy type

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References:


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