A new characterization of the Haagerup property by actions on infinite measure spaces. (English) Zbl 1475.22013 Ergodic Theory Dyn. Syst. 41, No. 8, 2349-2368 (2021).

A locally compact, second-countable topological group $G$ is said to have the Haagerup property if the constant function 1 on $G$ can be approximated, uniformly on compact subsets, by a sequence of normalized, positive-definite, vanishing-at-infinity functions.

This paper gives a new dynamical characterization of the Haagerup property: $G$ has the Haagerup property if and only if $G$ acts by measure-preserving automorphisms of some measure space $(\Omega, B, \mu)$, such that for all subsets $A, B \in \Omega$ of finite measure one has \(\lim_{g \to \infty} \mu(gA \cap B) = 0\), and such that $L^\infty(\Omega)$ has an invariant mean. (For comparison, recall that $G$ is amenable if and only if $G$ admits a proper, regular-Borel-measure-preserving action on a locally compact space $\Omega$ such that $L^\infty(\Omega)$ has an invariant mean.)

If $G$ admits such an action, then it is not hard to deduce from existing characterizations that $G$ has the Haagerup property; all of the work here goes into proving the converse. The starting point is a characterization of the Haagerup property in terms of strongly mixing actions on probability spaces (cf. Theorem 2.2.2 in [P.-A. Cherix et al., Groups with the Haagerup property. Gromov's a-T-menability. Basel: Birkhäuser (2001; Zbl 1030.43002)]). The presentation is clear and admirably detailed.

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MSC:
22D10 Unitary representations of locally compact groups
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References:

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