The topology of Baumslag-Solitar representations.


This paper is concerned with the homotopy type of representation varieties of Baumslag-Solitar groups into complex linear algebraic groups.

Recall that given a finitely-generated group $\Gamma$, with a fixed finite generating set $S \subset \Gamma$, and a topological group $G$, then every group morphism $\rho: \Gamma \to G$ can be identified with a point in $G^S$ given by the restriction $\rho|_S$. In this way, $\text{Hom}(\Gamma, G)$ can be seen a closed subspace of $G^S$. If $G$ is a complex linear algebraic group, this identification endows $\text{Hom}(\Gamma, G)$ with the structure of a complex affine algebraic variety which is called the representation variety of $\Gamma$ into $G$.

Here, the authors consider the case of $\Gamma$ being one of the Baumslag-Solitar groups $BS(p,q) := \langle a, b \mid abp a^{-1} = bq \rangle$, and prove:

**Theorem 1.3.** Let $p$ and $q$ be nonzero relatively prime integers with distinct absolute values, and consider $\Gamma = BS(p,q)$. If $G$ is the group of complex points of a reductive algebraic group, and $K < G$ is a maximal compact subgroup, then $\text{Hom}(\Gamma, K) \subset \text{Hom}(\Gamma, G)$ is a strong deformation retract. In particular, the two spaces are homotopy equivalent.

The paper is nicely written and mostly self contained. So, after recalling the basics on reductive algebraic groups, the authors observe that $\text{Hom}(\Gamma, G)$ can be simultaneously endowed with compatible structures of an affine algebraic variety (when thinking of $G$ as an affine algebraic group) and a Hausdorff space (when $G$ is thought as a Lie group), and that these two structures are isomorphic, thus the choice of presentation is immaterial.

Then the authors use standard Kempf-Ness theory to obtain an intermediate deformation retract between $\text{Hom}(\Gamma, K)$ and $\text{Hom}(\Gamma, G)$: the Kempf-Ness set denoted by $\text{Hom}_{KN}(\Gamma, G)$. This Kempf-Ness set is later on described as the total space of a bundle over a manifold of finite abelian subgroups of $G$ which allow to define a retraction of $\text{Hom}_{KN}(\Gamma, G)$ onto the subspace of representations such that $\rho((b)) < K$. A careful analysis of each fiber in the bundle is what finally allows to construct the desired retraction of $\text{Hom}_{KN}(\Gamma, G)$ onto $\text{Hom}(\Gamma, K)$.

Reviewer: Antonio Viruel (Málaga)

MSC:

55P99 Homotopy theory
20F19 Generalizations of solvable and nilpotent groups
20C99 Representation theory of groups
20G20 Linear algebraic groups over the reals, the complexes, the quaternions

Keywords:

representation variety; Baumslag-Solitar group; solvable group; complex reductive algebraic group; maximal compact subgroup; deformation retraction

Full Text: DOI

References:


This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.