Azevedo, João; Shumyatsky, Pavel
On finiteness of some verbal subgroups in profinite groups. (English) Zbl 1477.20066

Let \( w = w(x_1, \ldots, x_k) \) be a group word, that is a nontrivial element of the free group on \( x_1, \ldots, x_k \), and \( G \) a group. Then, \( w \) can be viewed as a \( k \)-variable function defined on \( G \). The set of \( w \)-values in \( G \) is denoted \( G_w \) and the subgroup generated by \( G_w \) is denoted \( w(G) \). If \( G \) is a topological group then \( w(G) \) is taken to be the closed subgroup.

The concept of conciseness of a word \( w \) in a class of groups \( C \), that is whether the finiteness of \( G_w \) implies the finiteness of \( w(G) \) for all \( G \) in \( C \), has a long history. More recently, a variation of conciseness for profinite groups has been considered. A word \( w \) is strongly concise in a class \( C \) of profinite groups if \( |G_w| < 2^{\aleph_0} \) implies that \( w(G) \) is finite for all \( G \) in \( C \). In this article, the authors consider a closely related question. They show that, for several families of words, if you suppose \( G \) is a profinite group with \( |G_w| < 2^{\aleph_0} \) and \( w(G) \) generated by finitely many \( w \)-values, then \( w(G) \) is finite.

In the first theorem, the authors consider words of type \([y, n v^q] \) and \([v^q, n y] \) where \( v \) is the left normed commutator \([x_1, x_2, \ldots, x_k]\) and \([y, n x] = [y, x, \ldots, x]\) with \( x \) repeated \( n \) times and \( k, n \) and \( q \) all positive integers. The second theorem is more technical to state, but covers many families of words.

Reviewer: Rachel D. Camina (Cambridge)

MSC:
20F10 Word problems, other decision problems, connections with logic and automata (group-theoretic aspects)
20E18 Limits, profinite groups
20E26 Residual properties and generalizations; residually finite groups
20F45 Engel conditions

Keywords:
profinite groups; words; conciseness

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References:


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